

# Notes

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$$ \xdef\scal#1#2{\langle #1, #2 \rangle} \xdef\norm#1{\left\| \right. #1 \right\|}
\xdef\dist{\rho} \xdef\and{\&} \xdef\AND{\quad \and \quad} \xdef\brackets#1{\left\{ #1 \right\}}
\xdef\parc#1#2{\frac {\partial #1} {\partial #2}} \xdef\mtr#1{\begin{pmatrix} #1 \end{pmatrix}}
\xdef\bm#1{\boldsymbol{#1}} \xdef\mc#1{\mathcal{#1}}
\xdef\vv#1{\mathbf{#1}} \xdef\vvp#1{\pmb{#1}} \xdef\ve{\varepsilon} \xdef\l{\lambda}
\xdef\th{\vartheta} \xdef\alpha{\alpha} \xdef\vf{\varphi} \xdef\Tagged#1{\text{#1}}
\xdef>tagged*#1{\text{#1}} \xdef>tagEqHere#1#2{\href{#2#eq-#1}{\text{#1}}}
\xdef>tagDeHere#1#2{\href{#2#de-#1}{\text{#1}}} \xdef>tagEq#1{\href{\#eq-#1}{\text{#1}}}
\xdef>tagDe#1{\href{\#de-#1}{\text{#1}}} \xdef\T#1{\htmlld{eq-#1}{#1}}
\xdef\D#1{\htmlld{de-#1}{\vv{#1}}} \xdef\conv#1{\mathrm{conv}}, #1}
\xdef\cone#1{\mathrm{cone}}, #1} \xdef\aff#1{\mathrm{aff}}, #1} \xdef\lin#1{\mathrm{Lin}}, #1}
\xdef\span#1{\mathrm{span}}, #1} \xdef\O{\mathcal O} \xdef\ri#1{\mathrm{ri}}, #1}
\xdef\rd#1{\mathrm{r}\partial, #1} \xdef\interior#1{\mathrm{int}}, #1} \xdef\proj{\Pi}
\xdef\epi#1{\mathrm{epi}}, #1} \xdef\grad#1{\mathrm{grad}}, #1}
\xdef\gradT#1{\mathrm{grad}}^T #1} \xdef\gradx#1{\mathrm{grad}}_x #1}
\xdef\hess#1{\nabla^2, #1} \xdef\hessx#1{\nabla^2_x #1} \xdef\jacobx#1{D_x #1}
\xdef\jacob#1{D #1} \xdef\grads#1#2{\mathrm{grad}}_{#1} #2} \xdef\subdif#1{\partial #1}
\xdef\co#1{\mathrm{co}}, #1} \xdef\iter#1{\wedge^{#1}} \xdef\str{\wedge^*} \xdef\spv{\mathcal V}
\xdef\civ{\mathcal U} \xdef\other#1{\hat{#1}} \xdef\prox{\mathrm{prox}}
\xdef\sign#1{\mathrm{sign}}, #1} \xdef\brackets#1{\left( #1 \right)} $$
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Počítejme  $(Y - X \xi)^T (Y - X \xi) = (Y^T - \xi^T X^T) (Y - X \xi) = Y^T Y - Y^T X \xi - \xi^T X^T Y + \xi^T X^T X \xi$

a pak  $\frac{\partial}{\partial \xi}$  tohoto výrazu je

$0 - (Y^T X)^T - X^T Y + (X^T X + (X^T X)^T) \xi = 0 - X^T Y - X^T Y + (X^T X + X^T X) \xi = -2 X^T Y + 2 X^T X \xi$

“ Viz přednáška z [lineárních statistických modelů](#)

## Proximální operátor $\| \cdot \|_1$ -normy

Víme, že  $\arg \min_x \|x\|_1$  je řešení minimalizačního problému  $\min_x \left( \lambda \|x\|_1 + \frac{1}{2} \|x - v\|_2^2 \right)$

Uvědomme si, že  $\frac{\partial}{\partial x} \|x\|_1 = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \\ \text{any value in } [-1, 1] & \text{if } x = 0 \end{cases}$ , proto  $\frac{\partial}{\partial x} \left( \lambda \|x\|_1 + \frac{1}{2} \|x - v\|_2^2 \right) = 0$  iff  $\lambda \text{sign}(x) = v$

$x_i \neq 0$  a také  $\|\nabla \|\mathbf{x} - \mathbf{v}\|_2^2\| = \nabla \sum_{i=1}^n (x_i - v_i)^2 = 2 \begin{pmatrix} x_1 - v_1 \\ \vdots \\ x_n - v_n \end{pmatrix}$  Tedy  
 stacionární bod  $\hat{\mathbf{x}}$  našeho problému musí splňovat  $\lambda \begin{pmatrix} \frac{\partial}{\partial x_1} \\ \vdots \\ \frac{\partial}{\partial x_n} \end{pmatrix} \mathbf{x}$

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