

9. cvičení

a) IS pro β_i : $T_i = \frac{\hat{\beta}_i}{\sqrt{\hat{\sigma}} (\mathbf{X}^T \mathbf{X})^{-1}_{i,i}} \sim t(n-p)$

Pak $P\left(T_i \in \left[t_{\frac{\alpha}{2}(n-p)}, t_{1-\frac{\alpha}{2}(n-p)}\right]\right) = 1 - \alpha$
 $t_{\frac{\alpha}{2}(n-p)} \leq T_i \leq t_{1-\frac{\alpha}{2}(n-p)}$
 $\frac{\beta_i - \hat{\beta}_i}{\sqrt{\hat{\sigma}^2 (\mathbf{X}^T \mathbf{X})^{-1}_{i,i}}} \leq t_{1-\frac{\alpha}{2}(n-p)}$
Z čehož dostaneme $\beta_i \in \left(\hat{\beta}_i \pm t_{1-\frac{\alpha}{2}(n-p)} \sqrt{\hat{\sigma}^2 (\mathbf{X}^T \mathbf{X})^{-1}_{i,i}}\right)$

b) $T = \frac{\mathbf{a}^T \mathbf{\beta}}{\sqrt{\hat{\sigma}^2 \mathbf{a}^T (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{a}}} \sim t(n-p)$

$P\left(\mathbf{a}^T \mathbf{\beta} \pm t_{1-\frac{\alpha}{2}(n-p)} \sqrt{\hat{\sigma}^2 \mathbf{a}^T (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{a}}\right)$

A regresní přímka pro dívky bude mít tvar $y = \hat{\beta}_0 + \hat{\beta}_1 x$ Pro chlapce: $y = \hat{\beta}_0 + \hat{\beta}_2 + (\hat{\beta}_1 + \hat{\beta}_3) x$ IS pro $\beta_1 + \beta_3$, tedy $\mathbf{a} = (0, 1, 0, 1)$

d) Při počítání predikčního intervalu zohledňujeme chybu u "nového pozorování". Tedy odhad rozptylu je $\hat{\sigma}^2 \mathbf{x}^T (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{x} + \hat{\sigma}^2$
 $\implies T = \frac{\mathbf{x}^T \mathbf{\beta}}{\sqrt{\hat{\sigma}^2 (1 + \mathbf{x}^T (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{x})}} \sim t(n-p)$

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# Confidence interval
predict(..., interval = "confidence")

# Or
predict(..., interval = "prediction")
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