9. cvičení

a) IS pro β_i: $$ T_i = \frac {\hat{\beta_i}} {\sqrt{\hat{\sigma} (\pmb X^T \pmb X)^{-1}_{i,i}}} \sim t(n-p) $$

Pak $$ P\left(T_ i \in \left[t_{\frac \alpha 2}(n-p), t_{1 - \frac \alpha 2}(n-p)\right]\right) = 1 - \alpha $$ $$ t_{\frac \alpha 2}(n-p) \leq T_ i \leq t_ {1 - \frac \alpha 2}(n-p) $$ $$ t_{\frac \alpha 2}(n-p) \leq \frac {\beta_i - \hat{\beta_ i}} {\sqrt{\hat{\sigma}^2 (\pmb X^T \pmb X)^{-1}_ {i,i}}} \leq t_{1 - \frac \alpha 2}(n-p) $$ Z čehož dostaneme $$ \beta_ i \in \left(\hat{\beta_ i} \pm t_{1-\frac \alpha 2}(n-p) \sqrt{\hat{\sigma}^2 (\pmb X^T \pmb X)^{-1}_ {i,i}}\right) $$

b) $$ T = \frac {\pmb a^T \pmb \beta^T} {\sqrt{\hat{\sigma}^2 \pmb a^T (\pmb X^T \pmb X)^{-1} \pmb a}} \sim t(n-p) $$

$$ \left(\pmb a^T \pmb \beta \pm t_{1-\frac \alpha 2}(n-p) \sqrt{\hat{\sigma}^2 \pmb a^T (\pmb X^T \pmb X)^{-1} \pmb a}\right) $$

A regresní přímka pro dívky bude mít tvar $$ y = \hat \beta_0 + \hat \beta_1 x $$ Pro chlapce: $$ y = \hat \beta_0 + \hat \beta_2 + (\hat \beta_1 + \hat \beta_3) x $$ IS pro $\beta_1 + \beta_3$, tedy $\pmb a = (0, 1, 0, 1)$

d) Při počítání predikčního intervalu zohledňujeme chybu u "nového pozorování". Tedy odhad rozptylu je $$ \hat{\sigma}^2 \pmb x^T (\pmb X^T \pmb X)^{-1} \pmb x + \hat \sigma^2 \implies T = \frac {\pmb x^T \pmb \beta^T} {\sqrt{\hat{\sigma}^2(1 + \pmb x^T (\pmb X^T \pmb X)^{-1} \pmb x)}} \sim t(n-p) $$

```
# Confidence interval
predict(..., interval = "confidence")
# Or
predict(..., interval = "prediction")
```

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