10. cvičení

$$ \xdef\mcal#1{\mathcal{#1}} \xdef\scal#1#2{\langle #1, #2 \rangle} \xdef\N{\mathbb N} \xdef\R{\mathbb R} \xdef\Q{\mathbb{Q}} \xdef\Z{\mathbb{Z}} \xdef\D{\mathbb{D}} \xdef\bm#1{\boldsymbol{#1}} \xdef\vv#1{\mathbf{#1}} \xdef\vvp#1{\pmb{#1}} \xdef\floor#1{\lfloor #1 \rfloor} \xdef\ceil#1{\lceil #1 \rceil} \xdef\grad#1{\mathrm{grad} , #1} \xdef\ve{\varepsilon} \xdef\im#1{\mathrm{im}(#1)} \xdef\tr#1{\mathrm{tr}(#1)} \xdef\norm#1{\left\vert \left\vert #1 \right\vert\right\vert} \xdef\scal#1#2{\langle #1, #2 \rangle} \xdef\ex#1{\mathrm{E} ,\left(#1\right)} \xdef\exv#1{\mathrm{E}, \vv{#1}} \xdef\mtrx#1{\begin{pmatrix}#1\end{pmatrix}} $$

Scheffeho věta $$ P\left([\vv b^T (A\hat\beta - A \beta)]^2 \leq m F_{1 - \alpha}(m, n - p) \hat \sigma^2 \vv b^T A (\vv X^T \vv X)^{-1} A^T \vv b\right) = 1 - \alpha $$ $\forall b \in \R^m$, je-li matice A typu $m \times p$ plné hodnosti.

Příklad $$ Y_i = \beta_0 + \beta_1 \cdot \text{Height}_I + \beta_2 \cdot \text{Sex}_i + \beta_3 \cdot (\text{Height} + \text{Sex})_i + \ve_i, \quad \ve \sim N(0, \sigma^2) $$ a chceme zkonstruovat 95% PS pro chlapce a dívky

1) Napíšeme tvar reg. křivky

- d: $y = \hat \beta_0 + \hat\beta_1 x$
- ch: $y = \hat\beta_0 + \hat\beta_2 + (\hat \beta_1 + \hat \beta_3)x$

2) Zvolíme vhodný tvar $\vv b$ a A:

- d: $\vv b = \mtrx{1 \ x} \in \R^2$, pak $$ \mtrx{1 & x} \overbrace{\mtrx{1 & 0 & 0 & 0 \ 0 & 1 & 0 & 0}}^A \mtrx{\hat \beta_1 \ \hat \beta_2 \ \hat \beta_3 \ \hat \beta_4} $$
- ch: $\vv b = \mtrx{1 \ x}$, pak $$ \mtrx{1 & x} \overbrace{\mtrx{1 & 0 & 1 & 0 \ 0 & 1 & 0 & 1}}^A \mtrx{\hat \beta_1 \ \hat \beta_2 \ \hat \beta_3 \ \hat \beta_4} $$

Nejprve počítejme pro dívky, Označme $$ \vv b^T A = \vv x^T = (1, x, 0, 0) $$ **3)** Odvodíme tvar pásu spolehlivosti (PS) $$ P\left([\vv x^T \hat \beta - \underbrace{\vv x^T \beta}_ {y = \beta_0 + \beta_1 x}]^2 \leq 2 F_ {1 - \alpha}(2, n - 4) \sigma^2 \vv x^T (\vv X^T \vv X)^{-1} \vv x \right) = 1 - \alpha $$ kde y je náhodná proměnná. Upravujme

$$ P\left(|\vv x^T \hat \beta - y| \leq \sqrt{2 F_{1 - \alpha}(2, n - 4) \sigma^2 \vv x^T (\vv X^T \vv X)^{-1} \vv x} \right) = 1 - \alpha $$

- pro $\vv x^T \hat \beta - y > 0$ dostáváme **dolní hranici** $$ P\left(y \geq \vv x^T \hat \beta - \sqrt{2 F_{1 - \alpha}(2, n - 4) \sigma^2 \vv x^T (\vv X^T \vv X)^{-1} \vv x} \right) = 1 - \alpha $$
- nebo pro $\vv x^T \hat \beta - y < 0$ dostáváme **horní hranici** $$ P\left(y \leq \vv x^T \hat \beta + \sqrt{2 F_{1 - \alpha}(2, n - 4) \sigma^2 \vv x^T (\vv X^T \vv X)^{-1} \vv x}

\right) = 1 - \alpha $$

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