

10. cvičení

$$\begin{aligned} & \text{\$}\text{\$} \quad \backslash\mathrm{mcal}\#1\{\backslash\mathrm{mathcal}\{ \#1\}\} \quad \backslash\mathrm{xscal}\#1\#2\{\backslash\angle \#1, \#2 \rangle\} \quad \backslash\mathrm{N}\{\backslash\mathrm{mathbb N}\} \\ & \backslash\mathrm{R}\{\backslash\mathrm{mathbb R}\} \quad \backslash\mathrm{Q}\{\backslash\mathrm{mathbb Q}\} \quad \backslash\mathrm{Z}\{\backslash\mathrm{mathbb Z}\} \quad \backslash\mathrm{D}\{\backslash\mathrm{mathbb D}\} \\ & \backslash\mathrm{bm}\#1\{\backslash\mathrm{boldsymbol}\{ \#1\}\} \quad \backslash\mathrm{vv}\#1\{\backslash\mathrm{mathbf}\{ \#1\}\} \quad \backslash\mathrm{vvp}\#1\{\backslash\mathrm{pmb}\{ \#1\}\} \\ & \backslash\mathrm{floor}\#1\{\backslash\mathrm{lfloor} \#1 \rfloor\} \quad \backslash\mathrm{ceil}\#1\{\backslash\mathrm{lceil} \#1 \rceil\} \quad \backslash\mathrm{grad}\#1\{\backslash\mathrm{mathrm}\{\mathrm grad\}, \#1\} \\ & \backslash\mathrm{ve}\{\backslash\mathrm{varepsilon}\} \quad \backslash\mathrm{im}\#1\{\backslash\mathrm{mathrm}\{\mathrm im\}(\#1)\} \quad \backslash\mathrm{tr}\#1\{\backslash\mathrm{mathrm}\{\mathrm tr\}(\#1)\} \\ & \backslash\mathrm{norm}\#1\{\backslash\mathrm{left}\backslash\mathrm{vert} \backslash\mathrm{left}\backslash\mathrm{vert} \#1 \backslash\mathrm{right}\backslash\mathrm{vert}\backslash\mathrm{right}\backslash\mathrm{vert}\} \quad \backslash\mathrm{xscal}\#1\#2\{\backslash\angle \#1, \#2 \rangle\} \\ & \backslash\mathrm{ex}\#1\{\backslash\mathrm{mathrm}\{\mathrm E\}, \backslash\mathrm{left}(\#1\backslash\mathrm{right})\} \quad \backslash\mathrm{exv}\#1\{\backslash\mathrm{mathrm}\{\mathrm E\}, \backslash\mathrm{vv}\{ \#1\}\} \\ & \backslash\mathrm{mtrx}\#1\{\backslash\mathrm{begin}\{\mathrm pmatrix\} \#1\backslash\mathrm{end}\{\mathrm pmatrix\}\} \text{\$}\text{\$} \end{aligned}$$

Scheffeho věta $\sum_{j=1}^m \left(\sum_{i=1}^p b_{ij}^2 (A_{ij} - \hat{A}_{ij})^2 \right) \leq m F_{1-\alpha}(m, n-p)$ $\hat{\sigma}^2 \sum_{j=1}^m \left(\sum_{i=1}^p b_{ij}^2 A_{ij} \sum_{i=1}^p X_i^2 \right) A_{ij} = 1 - \alpha$ $\forall b \in \mathbb{R}^m$, je-li matice A typu $m \times p$ plně hodnosti.

Příklad
$$Y_i = \beta_0 + \beta_1 \cdot \text{Height}_i + \beta_2 \cdot \text{Sex}_i + \beta_3 \cdot (\text{Height}_i + \text{Sex}_i)_i + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma^2)$$
 a chceme zkonstruovat 95% PS pro chlapce a dívky

1) Napíšeme tvar reg. křivky

- d: $y = \hat{\beta}_0 + \hat{\beta}_1 x$
- ch: $y = \hat{\beta}_0 + \hat{\beta}_2 + (\hat{\beta}_1 + \hat{\beta}_3)x$

2) Zvolíme vhodný tvar v a A :

- d: $\forall v \, b = \text{mtrix}\{1 \mid x\} \mid n \in \mathbb{R}^2$, pak $\overbrace{\text{mtrix}\{1 \& x\}}^{\text{mtrix}\{1 \& 0 \& 0 \& 0 \& 1 \& 0 \& 0\}}^A \text{mtrix}\{\hat{\beta}_1 \mid \hat{\beta}_2 \mid \hat{\beta}_3 \mid \hat{\beta}_4\}$
- ch: $\forall v \, b = \text{mtrix}\{1 \mid x\}$, pak $\overbrace{\text{mtrix}\{1 \& x\}}^{\text{mtrix}\{1 \& 0 \& 1 \& 0 \& 0 \& 1 \& 0 \& 1\}}^A \text{mtrix}\{\hat{\beta}_1 \mid \hat{\beta}_2 \mid \hat{\beta}_3 \mid \hat{\beta}_4\}$

Nejprve počítejme pro dívky, Označme $\mathbf{b}^T \mathbf{A} = \mathbf{x}^T = (1, x, 0, 0)$ **3)** Odvodíme tvar pásu spolehlivosti (PS)
$$P(\|\mathbf{x}^T \hat{\boldsymbol{\beta}} - \underbrace{\mathbf{x}^T \boldsymbol{\beta}}_{y = \beta_0 + \beta_1 x}\|^2 \leq 2 F_{\{1 - \alpha\}}(2, n - 4) \sigma^2 \mathbf{x}^T (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{x}) = 1 - \alpha$$
 kde y je náhodná proměnná. Upravujme

$$P\left(\|\hat{\beta} - y\| \leq \sqrt{2 F_{1-\alpha}(2, n-4)} \sigma \|\hat{X}^T \hat{X}\|^{-1} \|x\| \right) = 1 - \alpha$$

- pro $\|\mathbf{v}\| \hat{\beta} - y > 0$ dostáváme **dolní hranici** $P\left(y \geq \|\mathbf{v}\| \hat{\beta} - \sqrt{2 F_{1-\alpha}(2, n-4)} \sigma \|\mathbf{v}\| \sqrt{\mathbf{X}^T \mathbf{X}}^{-1} \|\mathbf{v}\| \right) = 1 - \alpha$
- nebo pro $\|\mathbf{v}\| \hat{\beta} - y < 0$ dostáváme **horní hranici** $P\left(y \leq \|\mathbf{v}\| \hat{\beta} + \sqrt{2 F_{1-\alpha}(2, n-4)} \sigma \|\mathbf{v}\| \sqrt{\mathbf{X}^T \mathbf{X}}^{-1} \|\mathbf{v}\| \right)$

$$\backslash\mathrm{right}) = 1 - \backslash\mathrm{alpha} \$\$$$

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