

# 10. cvičení

$$\begin{aligned} & \text{\$}\text{\$} \backslash\mathrm{mcal}\#1\{\backslash\mathrm{mathcal}\{ \#1\}\} \backslash\mathrm{sca}\#1\#2\{\backslash\angle \#1, \#2 \rangle\} \backslash\mathrm{N}\{\backslash\mathrm{mathbb N}\} \\ & \backslash\mathrm{R}\{\backslash\mathrm{mathbb R}\} \backslash\mathrm{Q}\{\backslash\mathrm{mathbb Q}\} \backslash\mathrm{Z}\{\backslash\mathrm{mathbb Z}\} \backslash\mathrm{D}\{\backslash\mathrm{mathbb D}\} \\ & \backslash\mathrm{bm}\#1\{\backslash\mathrm{boldsymbol}\{ \#1\}\} \backslash\mathrm{vv}\#1\{\backslash\mathrm{mathbf}\{ \#1\}\} \backslash\mathrm{vvp}\#1\{\backslash\mathrm{pmb}\{ \#1\}\} \\ & \backslash\mathrm{floor}\#1\{\backslash\mathrm{lfloor} \#1 \rfloor\} \backslash\mathrm{ceil}\#1\{\backslash\mathrm{lceil} \#1 \rceil\} \backslash\mathrm{grad}\#1\{\backslash\mathrm{mathrm}\{\mathrm grad\}, \#1\} \\ & \backslash\mathrm{ve}\{\backslash\mathrm varepsilon\} \backslash\mathrm{im}\#1\{\backslash\mathrm{mathrm}\{\mathrm im\}( \#1)\} \backslash\mathrm{tr}\#1\{\backslash\mathrm{mathrm}\{\mathrm tr\}( \#1)\} \\ & \backslash\mathrm{norm}\#1\{\left|\right.\mathrm{vert} \left|\right.\mathrm{vert} \#1 \right|\right.\mathrm{vert} \right|\mathrm{right}\mathrm{vert}\} \backslash\mathrm{sca}\#1\#2\{\backslash\angle \#1, \#2 \rangle\} \\ & \backslash\mathrm{ex}\#1\{\backslash\mathrm{mathrm}\{\mathrm E\}, \left( \#1\right)\} \backslash\mathrm{exv}\#1\{\backslash\mathrm{mathrm}\{\mathrm E\}, \mathrm vv\{ \#1\}\} \\ & \backslash\mathrm{mtrx}\#1\{\backslash\mathrm{begin}\{\mathrm pmatrix\} \#1\backslash\mathrm{end}\{\mathrm pmatrix\}\} \text{\$}\text{\$} \end{aligned}$$

**Scheffeho věta**  $\sum_{b \in \mathbb{R}^m} P\left(\|\sum_{v \in V} b^T (A \hat{\beta} - A \beta)\|^2 \leq m F_{1-\alpha}(m, n-p)\right) = 1 - \alpha$   $\forall b \in \mathbb{R}^m$ , je-li matice  $A$  typu  $m \times p$  plně hodnosti.

Příklad 
$$Y_i = \beta_0 + \beta_1 \cdot \text{Height}_i + \beta_2 \cdot \text{Sex}_i + \beta_3 \cdot (\text{Height}_i + \text{Sex}_i)_i + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma^2)$$
 a chceme zkonstruovat 95% PS pro chlapce a dívky

**1) Napíšeme tvar reg. křivky**

- d:  $y = \hat{\beta}_0 + \hat{\beta}_1 x$
- ch:  $y = \hat{\beta}_0 + \hat{\beta}_2 + (\hat{\beta}_1 + \hat{\beta}_3)x$

**2)** Zvolíme vhodný tvar  $v$  a  $A$ :

- d:  $\forall v \, b = \begin{pmatrix} 1 & x \end{pmatrix} \in \mathbb{R}^2$ , pak  $\overbrace{\begin{pmatrix} 1 & x \end{pmatrix}}^{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}^A} \begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \hat{\beta}_4 \end{pmatrix}$
- ch:  $\forall v \, b = \begin{pmatrix} 1 & x \end{pmatrix}$ , pak  $\overbrace{\begin{pmatrix} 1 & x \end{pmatrix}}^{\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}^A} \begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \hat{\beta}_4 \end{pmatrix}$

Nejprve počítejme pro dívky, Označme  $\mathbb{V} b^T A = \mathbb{V} x^T = (1, x, 0, 0)$  **3)** Odvodíme tvar pásu spolehlivosti (PS) 
$$P(\| \mathbb{V} x^T \hat{\beta} - \underbrace{\mathbb{V} x^T \beta}_y \| \leq \beta_0 + \beta_1 x)^2 \leq 2 F_{1-\alpha}(2, n-4) \sigma^2 \mathbb{V} x^T (\mathbb{V} X^T \mathbb{V} X)^{-1} \mathbb{V} x$$
 kde  $y$  je náhodná proměnná. Upravujeme

$$P\left(\|\hat{\beta} - y\| \leq \sqrt{2 F_{1-\alpha}(2, n-4)} \sigma \|\hat{X}^T \hat{X}\|^{-1} \|x\| \right) = 1 - \alpha$$

- pro  $\forall x \in T \text{ : } \hat{\beta} - y > 0$  dostáváme **dolní hranici**  $P\left(y \geq \forall x \in T \text{ : } \hat{\beta} - \sqrt{2 F_{1-\alpha}(2, n-4)} \sigma \mid \forall x \in T \text{ : } (\forall X \in \forall X)^{-1} \mid \forall x\right) = 1 - \alpha$
- nebo pro  $\forall x \in T \text{ : } \hat{\beta} - y < 0$  dostáváme **horní hranici**  $P\left(y \leq \forall x \in T \text{ : } \hat{\beta} + \sqrt{2 F_{1-\alpha}(2, n-4)} \sigma \mid \forall x \in T \text{ : } (\forall X \in \forall X)^{-1} \mid \forall x\right)$

$$\backslash\mathrm{right}) = 1 - \backslash\mathrm{alpha} \$\$$$

---

Revision #2

Created 12 January 2023 12:12:00 by Sceptri

Updated 12 January 2023 12:57:01 by Sceptri