

# 10. cvičení

$\$ \xdef\mcal#1{\mathcal{#1}} \xdef\scal#1#2{\langle #1, #2 \rangle} \xdef\N{\mathbb N}$   
 $\xdef\R{\mathbb R} \xdef\Q{\mathbb Q} \xdef\Z{\mathbb Z} \xdef\D{\mathbb D}$   
 $\xdef\bm#1{\boldsymbol{#1}} \xdef\vv#1{\mathbf{#1}} \xdef\vvp#1{\pmb{#1}}$   
 $\xdef\floor#1{\lfloor #1 \rfloor} \xdef\ceil#1{\lceil #1 \rceil} \xdef\grad#1{\mathrm{grad} , #1}$   
 $\xdef\ve{\mathrm{varepsilon}} \xdef\im#1{\mathrm{im}(#1)} \xdef\tr#1{\mathrm{tr}(#1)}$   
 $\xdef\norm#1{\left\| \right\|_{#1}} \xdef\scal#1#2{\langle #1, #2 \rangle}$   
 $\xdef\ex#1{\mathrm{E} , \left( #1 \right)} \xdef\exv#1{\mathrm{E} , \vv{#1}}$   
 $\xdef\mtrx#1{\begin{pmatrix} #1 \end{pmatrix}} \$$

**Scheffeho věta**  $P\left(\|\vv{b}^T (A\hat{\beta} - A\beta)\|^2 \leq m F_{1-\alpha}(m, n-p) \hat{\sigma}^2 \mid \vv{b}^T A (\vv{X}^T \vv{X})^{-1} A^T \vv{b}\right) = 1 - \alpha$   $\forall b \in \mathbb{R}^m$ , je-li matice  $A$  typu  $m \times p$  plně hodnosti.

Příklad  $Y_i = \beta_0 + \beta_1 \text{Height}_i + \beta_2 \text{Sex}_i + \beta_3 \text{Height}_i \cdot \text{Sex}_i + \epsilon_i$ ,  $\epsilon_i \sim N(0, \sigma^2)$  a chceme zkonstruovat 95% PS pro chlapce a dívky

**1)** Napíšeme tvar reg. křivky

- d:  $y = \hat{\beta}_0 + \hat{\beta}_1 x$
- ch:  $y = \hat{\beta}_0 + \hat{\beta}_2 + (\hat{\beta}_1 + \hat{\beta}_3)x$

**2)** Zvolíme vhodný tvar  $\vv{b}$  a  $A$ :

- d:  $\vv{b} = \begin{pmatrix} 1 & x \end{pmatrix} \in \mathbb{R}^2$ , pak  $\overbrace{\begin{pmatrix} 1 & x \end{pmatrix} A^T A \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}}^{\hat{\beta}_1 \ \hat{\beta}_2 \ \hat{\beta}_3 \ \hat{\beta}_4}$
- ch:  $\vv{b} = \begin{pmatrix} 1 & x \end{pmatrix}$ , pak  $\overbrace{\begin{pmatrix} 1 & x \end{pmatrix} A^T A \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}}^{\hat{\beta}_1 \ \hat{\beta}_2 \ \hat{\beta}_3 \ \hat{\beta}_4}$

Nejprve počítejme pro dívky, Označme  $\vv{b}^T A = \vv{x}^T = (1, x, 0, 0)$  **3)** Odvodíme tvar pásu spolehlivosti (PS)  $P\left(\|\vv{x}^T \hat{\beta} - \underbrace{\vv{x}^T \beta}_{y = \beta_0 + \beta_1 x}\|^2 \leq 2 F_{1-\alpha}(2, n-4) \hat{\sigma}^2 \mid \vv{x}^T (\vv{X}^T \vv{X})^{-1} \vv{x} \right) = 1 - \alpha$  kde  $y$  je náhodná proměnná. Upravujeme

$P\left(\|\vv{x}^T \hat{\beta} - y\| \leq \sqrt{2 F_{1-\alpha}(2, n-4) \hat{\sigma}^2 \mid \vv{x}^T (\vv{X}^T \vv{X})^{-1} \vv{x}} \right) = 1 - \alpha$

- pro  $\vv{x}^T \hat{\beta} - y > 0$  dostáváme **dolní hranici**  $P(y \geq \vv{x}^T \hat{\beta} - \sqrt{2 F_{1-\alpha}(2, n-4) \hat{\sigma}^2 \mid \vv{x}^T (\vv{X}^T \vv{X})^{-1} \vv{x}}) = 1 - \alpha$
- nebo pro  $\vv{x}^T \hat{\beta} - y < 0$  dostáváme **horní hranici**  $P(y \leq \vv{x}^T \hat{\beta} + \sqrt{2 F_{1-\alpha}(2, n-4) \hat{\sigma}^2 \mid \vv{x}^T (\vv{X}^T \vv{X})^{-1} \vv{x}}) = 1 - \alpha$

$$\backslash\mathrm{right}) = 1 - \backslash\mathrm{alpha} \$\$$$

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