

10. cvičení

$\$ \xdef\mcal#1{\mathcal{#1}} \xdef\scal#1#2{\langle #1, #2 \rangle} \xdef\N{\mathbb N}$
 $\xdef\R{\mathbb R} \xdef\Q{\mathbb Q} \xdef\Z{\mathbb Z} \xdef\D{\mathbb D}$
 $\xdef\bm#1{\boldsymbol{#1}} \xdef\vv#1{\mathbf{#1}} \xdef\vp#1{\mathbf{#1}}$
 $\xdef\floor#1{\lfloor #1 \rfloor} \xdef\ceil#1{\lceil #1 \rceil} \xdef\grad#1{\mathrm{grad} , #1}$
 $\xdef\ve{\mathrm{varepsilon}} \xdef\im#1{\mathrm{im}(#1)} \xdef\tr#1{\mathrm{tr}(#1)}$
 $\xdef\norm#1{\left\| \right\|_{#1}} \xdef\scal#1#2{\langle #1, #2 \rangle}$
 $\xdef\ex#1{\mathrm{E} , \left(#1 \right)} \xdef\exv#1{\mathrm{E} , \mathbf{#1}}$
 $\xdef\mtrx#1{\begin{pmatrix} #1 \end{pmatrix}} \$$

Scheffeho věta $P\left(\|\mathbf{b}^T (\hat{\beta} - \beta)\|^2 \leq m F_{1-\alpha}(m, n-p) \hat{\sigma}^2 \mathbf{b}^T A (\mathbf{X}^T \mathbf{X})^{-1} A^T \mathbf{b}\right) = 1 - \alpha$ $\forall \mathbf{b} \in \mathbb{R}^m$, je-li matice A typu $m \times p$ plně hodnosti.

Příklad $Y_i = \beta_0 + \beta_1 \text{Height}_i + \beta_2 \text{Sex}_i + \beta_3 \text{Height}_i \text{Sex}_i + \epsilon_i$, $\epsilon_i \sim N(0, \sigma^2)$ a chceme zkonstruovat 95% PS pro chlapce a dívky

1) Napíšeme tvar reg. křivky

- d: $y = \hat{\beta}_0 + \hat{\beta}_1 x$
- ch: $y = \hat{\beta}_0 + \hat{\beta}_2 + (\hat{\beta}_1 + \hat{\beta}_3)x$

2) Zvolíme vhodný tvar \mathbf{b} a A :

- d: $\mathbf{b} = \begin{pmatrix} 1 \\ x \end{pmatrix} \in \mathbb{R}^2$, pak $\overbrace{\begin{pmatrix} 1 & x \end{pmatrix} A}^{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}} \hat{\beta} = \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{pmatrix}$
- ch: $\mathbf{b} = \begin{pmatrix} 1 \\ x \end{pmatrix}$, pak $\overbrace{\begin{pmatrix} 1 & x \end{pmatrix} A}^{\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}} \hat{\beta} = \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_2 \\ \hat{\beta}_1 \\ \hat{\beta}_3 \end{pmatrix}$

Nejprve počítejme pro dívky, Označme $\mathbf{b}^T A = \mathbf{x}^T = (1, x, 0, 0)$ **3)** Odvodíme tvar pásu spolehlivosti (PS) $P\left(\|\mathbf{x}^T \hat{\beta} - \underbrace{\mathbf{x}^T \beta}_{y = \beta_0 + \beta_1 x}\|^2 \leq 2 F_{1-\alpha}(2, n-4) \hat{\sigma}^2 \mathbf{x}^T (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{x}\right) = 1 - \alpha$ kde y je náhodná proměnná. Upravujeme

$P\left(\|\mathbf{x}^T \hat{\beta} - y\| \leq \sqrt{2 F_{1-\alpha}(2, n-4) \hat{\sigma}^2 \mathbf{x}^T (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{x}}\right) = 1 - \alpha$

- pro $\mathbf{x}^T \hat{\beta} - y > 0$ dostáváme **dolní hranici** $P(y \geq \mathbf{x}^T \hat{\beta} - \sqrt{2 F_{1-\alpha}(2, n-4) \hat{\sigma}^2 \mathbf{x}^T (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{x}}) = 1 - \alpha$
- nebo pro $\mathbf{x}^T \hat{\beta} - y < 0$ dostáváme **horní hranici** $P(y \leq \mathbf{x}^T \hat{\beta} + \sqrt{2 F_{1-\alpha}(2, n-4) \hat{\sigma}^2 \mathbf{x}^T (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{x}})$

$$\backslash\mathrm{right}) = 1 - \backslash\mathrm{alpha} \$\$$$

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