

10. cvičení

$$\begin{aligned} & \text{\$}\text{\$} \quad \backslash\mathrm{mcal}\#1\{\backslash\mathrm{mathcal}\{ \#1\}\} \quad \backslash\mathrm{xscal}\#1\#2\{\backslash\angle \#1, \#2 \rangle\angle\} \quad \backslash\mathrm{def}\N{\backslash\mathrm{mathbb N}} \\ & \backslash\mathrm{def}\R{\backslash\mathrm{mathbb R}} \quad \backslash\mathrm{def}\Q{\backslash\mathrm{mathbb}\{Q\}} \quad \backslash\mathrm{def}\Z{\backslash\mathrm{mathbb}\{Z\}} \quad \backslash\mathrm{def}\D{\backslash\mathrm{mathbb}\{D\}} \\ & \backslash\mathrm{def}\bm\#1\{\backslash\mathrm{boldsymbol}\{ \#1\}\} \quad \backslash\mathrm{def}\vv\#1\{\backslash\mathrm{mathbf}\{ \#1\}\} \quad \backslash\mathrm{def}\vvp\#1\{\backslash\mathrm{pmb}\{ \#1\}\} \\ & \backslash\mathrm{def}\floor\#1\{\backslash\mathrm{ifloor} \#1 \rfloor\} \quad \backslash\mathrm{def}\ceil\#1\{\backslash\mathrm{lceil} \#1 \rceil\} \quad \backslash\mathrm{def}\grad\#1\{\backslash\mathrm{mathrm}\{\mathrm grad\}, \#1\} \\ & \backslash\mathrm{def}\ve{\backslash\mathrm varepsilon} \quad \backslash\mathrm{def}\im\#1\{\backslash\mathrm{mathrm}\{\mathrm im\}(\#1)\} \quad \backslash\mathrm{def}\tr\#1\{\backslash\mathrm{mathrm}\{\mathrm tr\}(\#1)\} \\ & \backslash\mathrm{def}\norm\#1\{\left\|\right.\vert \left.\right\| \backslash\mathrm{right}\backslash\mathrm{vert}\backslash\mathrm{right}\backslash\mathrm{vert}\} \quad \backslash\mathrm{def}\scal\#1\#2\{\backslash\angle \#1, \#2 \rangle\angle\} \\ & \backslash\mathrm{def}\ex\#1\{\backslash\mathrm{mathrm}\{E\}, \left(\#1\right)\} \quad \backslash\mathrm{def}\exv\#1\{\backslash\mathrm{mathrm}\{E\}, \vv\{ \#1\}\} \\ & \backslash\mathrm{def}\mtrx\#1\{\backslash\mathrm{begin}\{\mathrm pmatrix\} \#1\backslash\mathrm{end}\{\mathrm pmatrix\}\} \quad \text{\$}\text{\$} \end{aligned}$$

Scheffeho věta $\sum_{j=1}^m \left(\sum_{i=1}^p b_{ij}^2 (\hat{\beta}_i - \beta_i)^2 \right) \leq m F_{1-\alpha}(m, n-p) \hat{\sigma}^2 \sum_{j=1}^m \sum_{i=1}^p b_{ij}^2 = 1-\alpha$ $\forall b \in \mathbb{R}^m$, je-li matice A typu $m \times p$ plně hodnosti.

Příklad $Y_i = \beta_0 + \beta_1 \cdot \text{Height}_i + \beta_2 \cdot \text{Sex}_i + \beta_3 \cdot (\text{Height} + \text{Sex})_i + \epsilon_i$, $\epsilon_i \sim N(0, \sigma^2)$ a chceme
zkonstruovat 95% PS pro chlapce a dívky

1) Napíšeme tvar reg. křivky

- d: $y = \hat{\beta}_0 + \hat{\beta}_1 x$
- ch: $y = \hat{\beta}_0 + \hat{\beta}_2 + (\hat{\beta}_1 + \hat{\beta}_3)x$

2) Zvolíme vhodný tvar v a A :

- d: $\forall v \, b = \text{mtrix}\{1 \mid x\} \mid n \in \mathbb{R}^2$, pak $\overbrace{\text{mtrix}\{1 \& x\}}^{\text{mtrix}\{1 \& 0 \& 0 \& 0 \& 1 \& 0 \& 0\}}^A \text{mtrix}\{\hat{\beta}_1 \mid \hat{\beta}_2 \mid \hat{\beta}_3 \mid \hat{\beta}_4\}$
- ch: $\forall v \, b = \text{mtrix}\{1 \mid x\}$, pak $\overbrace{\text{mtrix}\{1 \& x\}}^{\text{mtrix}\{1 \& 0 \& 1 \& 0 \& 0 \& 1 \& 0 \& 1\}}^A \text{mtrix}\{\hat{\beta}_1 \mid \hat{\beta}_2 \mid \hat{\beta}_3 \mid \hat{\beta}_4\}$

Nejprve počítejme pro dívky, Označme $\mathbf{b}^T \mathbf{A} = \mathbf{x}^T = (1, x, 0, 0)$ **3)** Odvodíme tvar pásu spolehlivosti (PS)
$$P(\|\mathbf{x}^T \hat{\mathbf{\beta}} - \underbrace{\mathbf{x}^T \mathbf{\beta}}_{y = \beta_0 + \beta_1 x}\|^2 \leq 2 F_{\{1 - \alpha\}}(2, n - 4) \sigma^2 \mathbf{x}^T (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{x}) = 1 - \alpha$$
 kde y je náhodná proměnná. Upravujme

$$P\left(\|\hat{\beta} - y\| \leq \sqrt{2 F_{1-\alpha}(2, n-4)} \sigma \|\hat{X}^T \hat{X}\|^{-1} \|x\| \right) = 1 - \alpha$$

- pro $\|\mathbf{v}\| \hat{\beta} - y > 0$ dostáváme **dolní hranici** $P\left(y \geq \|\mathbf{v}\| \hat{\beta} - \sqrt{2 F_{1-\alpha}(2, n-4)} \sigma \|\mathbf{v}\| \sqrt{\mathbf{X}^T \mathbf{X}}^{-1} \|\mathbf{v}\| \right) = 1 - \alpha$
- nebo pro $\|\mathbf{v}\| \hat{\beta} - y < 0$ dostáváme **horní hranici** $P\left(y \leq \|\mathbf{v}\| \hat{\beta} + \sqrt{2 F_{1-\alpha}(2, n-4)} \sigma \|\mathbf{v}\| \sqrt{\mathbf{X}^T \mathbf{X}}^{-1} \|\mathbf{v}\| \right)$

$$\backslash\mathrm{right}) = 1 - \backslash\mathrm{alpha} \$\$$$

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